

Study of Exchange of Brines through Dykes with Viscous Flow Analogy

Anching Lin
Sang-Myung Lee

Department of Civil Engineering
University of Utah, Salt Lake City, Utah

ABSTRACT

The method of viscous flow analogy is introduced for the solution of two-phase fluid flows in porous media. A model study is applied to a case study, the investigation of the exchange of brines through the causeway of the Great Salt Lake. The method is recommended as a general method for the study of seepage problems encountered in the production of salt.

INTRODUCTION

Seepage losses of brines through dykes or similar structures have been a major consideration in the design and construction of transportation ditches and retention ponds for the production of salt. Although theory of seepage has been developed for some time, few scientists or engineers have directed their attention to specific problems pertinent to salt industries.

Typical seepage problems in the production of salt would involve the intrusion of heavier brines into lighter ones or even fresh water. In general, this type of problem can be classified as two-phase fluid flow in porous media, which is normally difficult to handle with analytical means. The classical exposition by Palubarinova-Kochina (1962) introduced various methods and presented an extensive collection of analytical solutions to the single-phase fluid flow problems in porous media. With an emphasis on two-phase fluid flows, this paper describes a method, called viscous flow analogy (Lin and Lee, 1972) which has been widely used among the fluid dynamicists for the solution of potential flows, to which the seepage flow through a porous media belongs.

The viscous flow analogy allows one to study flows through two dimensional porous media, e.g., dykes and earth dams, with a Hele-Shaw cell on which the quantitative measurements are more practicable than on the prototypes. The limitations of the method are: (1) the geometry

must be such that the resulting flows are two dimensional; (2) the lumped soil properties of the porous media must be homogeneous. Fortunately, for flows through most man-made structures, these limitations do not present serious problems, and the method is found to be applicable in most practical instances. The method will be discussed by way of a case study on seepage flow through the railroad causeway on the Great Salt Lake.

THE HELE-SHAW METHOD

Hele-Shaw, in 1898, suggested the use of viscous flow analogy to study the potential flows (Todd, 1954). The analogy is made possible by establishing the similarity of the dynamic equations of motion between the flow through porous media and the flow in a very thin gap called a Hele-Shaw cell. That is, if one builds a Hele-Shaw model which is geometrically similar to a prototype, e.g., the shape of the cross-section of dykes, the flows through the model and the prototype are similar and conditions in the model can be interpreted and utilized in the prototype if the proper modelling laws are acknowledged (Deweist, 1965; Bear, 1960).

$$\lambda_K \sim \frac{\lambda_L}{\lambda_T}$$

where λ_K , λ_L , λ_T are the ratios of permeability, length and time between the model and the prototype. Here the permeability of the model is defined as $K_m = ga^2/3\nu$, a being the thickness of the Hele-Shaw cell and ν the viscosity of liquid used in the model.

A CASE STUDY

The Southern Pacific Railroad causeway in the Great Salt Lake is a rockfilled structure. Over the years, new materials have been added to the original structure to

stabilize the slope. Thus the causeway is composed mainly of two parts: (a) the core portion consisting of more compact materials, and (b) the cover portion on the slope resulting from newly added material and through which water flows with little resistance. It is the core portion that is responsible for the dynamic consequence of the causeway and thus the modelling is focused on the core portion.

A Hele-Shaw cell was built to the estimated proportions of the core portion of the causeway with a side slope of 1.5 units in horizontal direction versus 1.0 unit in the vertical. The overall base dimension of the model is about 24 inches and the working thickness of the Hele-Shaw cell was chosen to be $\frac{1}{8}$ inch. The cell was constructed with transparent plexiglas for the convenience of visualization.

The presence of two different brines on two sides of the causeway created the two-phase seepage flow through the bottom of one side to the other side of the causeway, and an opposite flow of the lighter brines on the upper layer (Adams, 1964; Madison, 1970).

The steady-state situation of the two phase seepage flow in the causeway was simulated in the model by maintaining a constant head difference through pumping by two polystaltic pumps. Dye materials were introduced to track the seepage motion. The movement of liquid in the model (as well as in the prototype) is typically very slow, therefore, the model study was carried out by first completely recording the fluid motion in 16mm movies with a time lapse device and then each movie frame was examined to extract such information as velocity distribution and seepage volumes.

DISCUSSION

The most important parameters which determine the seepage characteristics are the density ratios of the two brines and the relative liquid-head difference between the two sides of the structure. It was observed, as expected, that there were two extremes that might occur: i.e., when head difference was too small or too large, the seepage flow would involve the one-way movement of either lighter liquid or heavier liquid without exchange between the two.

Normally, the flow takes place between the extremes. Depending on the density ratio and relative head differences, there are exchanges of fluids between the two sides of the model. And the result of such observations can best be summarized in Figure 1. Note that the subscripts 1 and 2 are associated with conditions to the left and to the right, respectively, of the model structure (or the south arm and the north arm of the actual causeway). The curve was developed for $\rho_1/\rho_2 = 0.777$. From the curve, one could predict the ratios of rates of seepage volume Q_1/Q_2 for a given relative head difference $\Delta H/H_1$. Figures 2 through 5 give the actual steady state configuration of the five runs performed and also define the symbols mentioned in the study. Figures 2, 3 and 4 demonstrate the fact that there are exchanges of liquids between the two sides of the

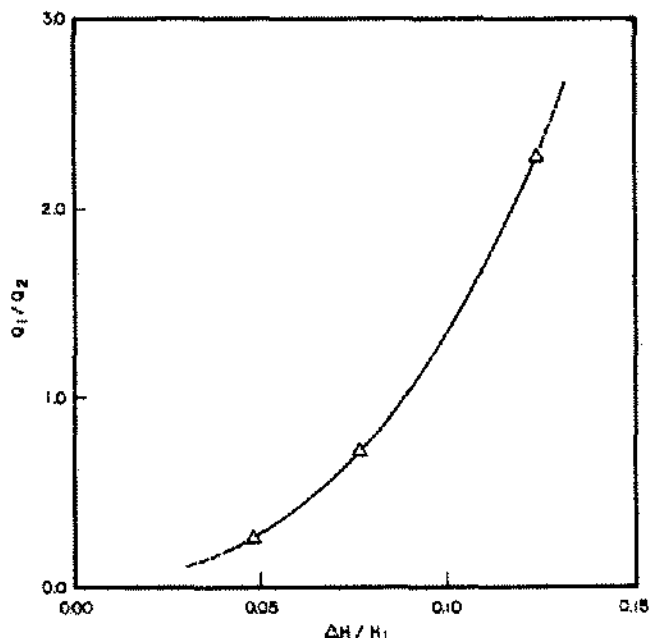


Figure 1. Ratio of Seepage Volumes Versus Density Ratios.

structure. Figure 5 indicates the case, due to excessive head differences, when there is only one-way migration of the lighter liquid.

The velocity vectors shown are the results of study taken on movie film. The integration of the velocity profiles then gives the rates of seepage volume. The results presented in dimensionless form such as Q_1/Q_2 , $\Delta H/H_1$, etc., are readily applicable to the prototype without having to consider the modelling laws.

One may consider the year 1972 as an illustration. The head difference between the south arm and the north arm of the Lake has been quoted as 1.8 feet and the water depth at the south side of the causeway averages, say, 25 feet. Then, the relative head difference $\Delta H/H_1$ is estimated to be 0.07 (or 7%). If the density ratio were 0.777, then the ratio of exchange volume Q_1/Q_2 would have been about 0.50 (Fig. 1). Or equivalently, if the density of the brines in the south were diluted to 78% of that in the north, then relative head difference 0.07 would have indicated that heavier brines were migrating two times as much from the north to the south than the much diluted brines from the south to the north. Of course, the actual density ratio is about 0.90 and the ratio of seepage volumes will be higher than 0.50.

From a consideration similar to Ghyben-Herzberg's (Dewiest, 1965), and working from Figure 1 the ratio of seepage volume should be close to one. That is, the volumes of brines migrated from the south are almost balanced by those from the north.

The model study offers a simple and direct approach to the investigation of the dynamic behavior of seepage flow through the man-made structures. It is by far the most

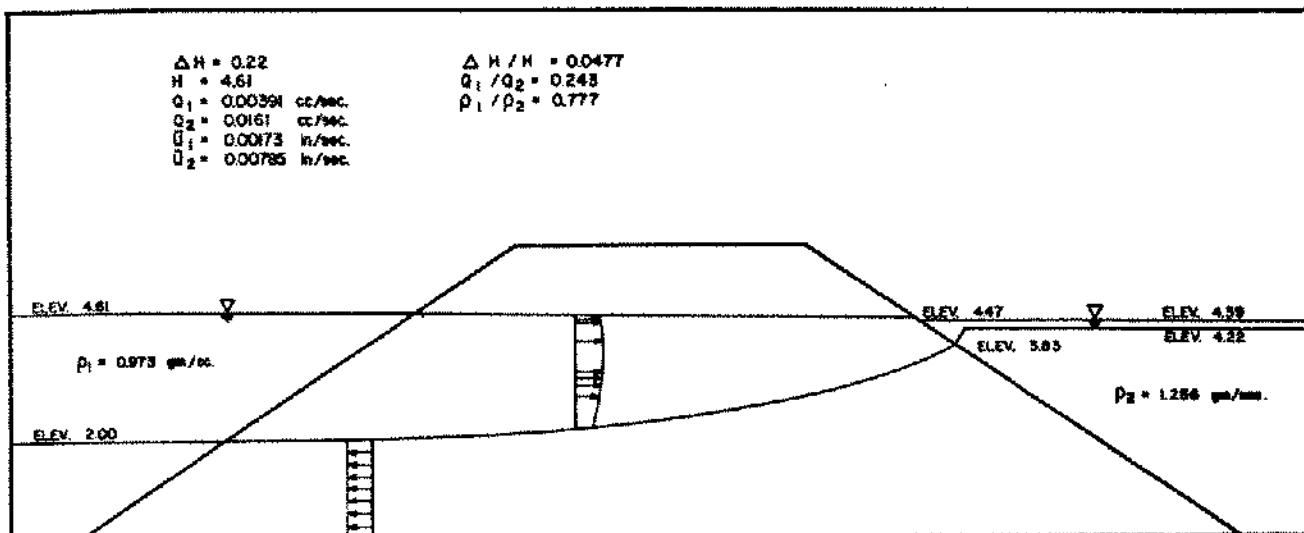


Figure 2. Flow Configuration I.

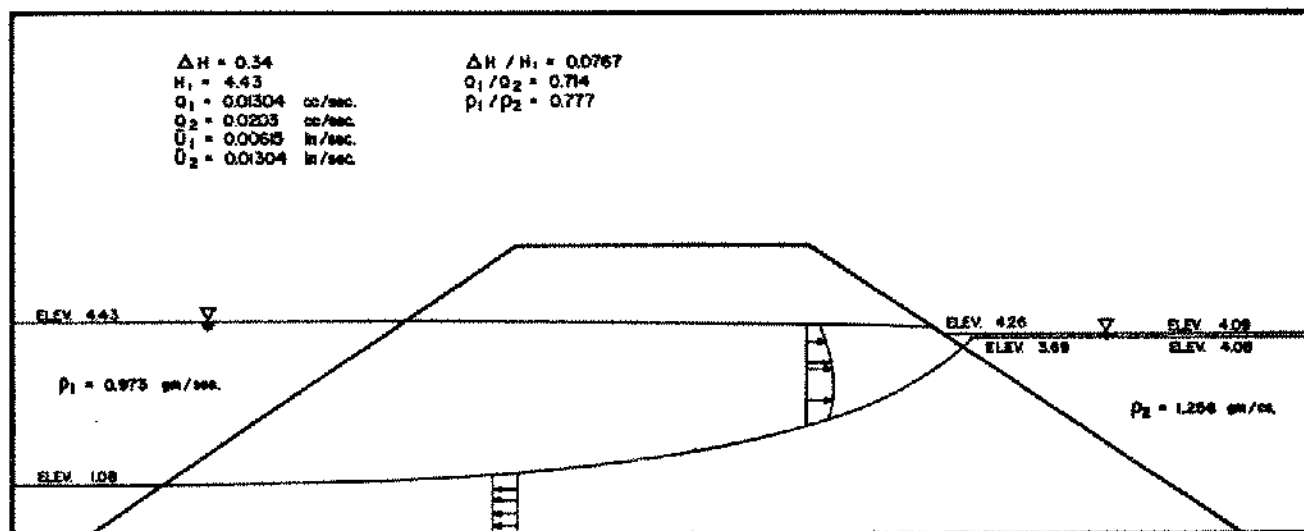


Figure 3. Flow Configuration II.

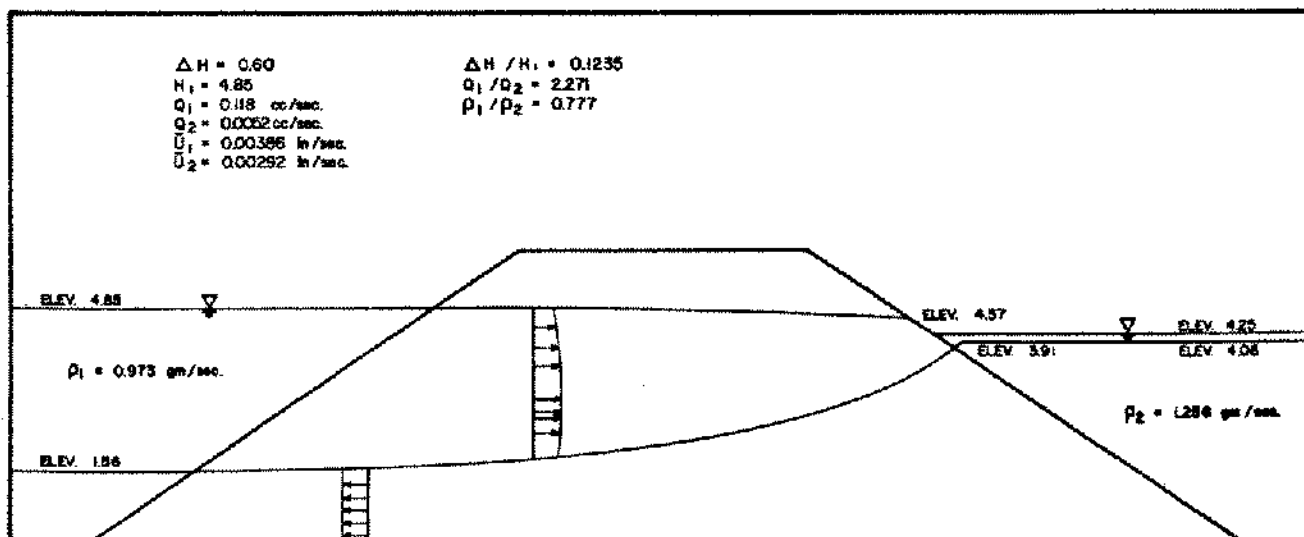


Figure 4. Flow Configuration III.

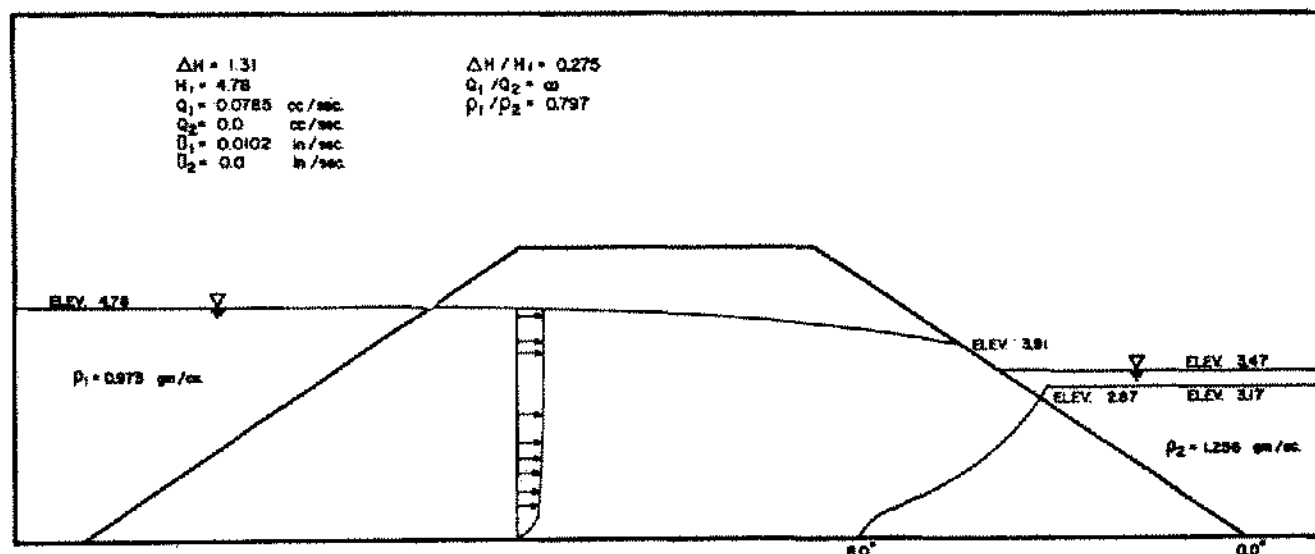


Figure 5. Flow Configuration IV.

effective and economical way for the treatment of seepage flows involving the exchange of two-phase fluids.

Coupled with field investigations, the model study should provide all the information needed for engineering designs. It will furnish comprehensive details of the seepage parameters, such as velocity distributions, exchange of seepage volumes, locus of interface, etc. The analogy is especially useful if the problem involves time-dependent boundary conditions such as fluctuations resulting from tidal waves or other wave actions.

ACKNOWLEDGMENT

This work is partially supported by the Utah Engineering Experiment Station, by the Development Fund from the College of Engineering and by Morton Salt Research Laboratory. Materials related to the causeway study were presented in the "Symposium on the Great Salt Lake and Utah's Water Resources," (Lin and Lee, 1972).

REFERENCES

- Adams, T. C., 1964, Salt Migration to the Northwest Body of Great Salt Lake, Utah, *Science*, Vol. 143, Pages 1027-1029.
- Bear, J., 1960, Scales of Viscous Analogy Models for Ground Water Studies, *American Society of Civil Engineers, J. of Hydr. Div.*, Vol. 86, HY2 Pages 11-24.
- DeWiest, R. J. M., 1965, *Geohydrology*, John Wiley, New York, Pages 322-331.
- Madison, R. J., 1970, Effect of a Causeway on the Chemistry of the Brine in Great Salt Lake, Utah: *Utah Geological and Mineralogical Survey Water Resources Bulletin*, 14, 52 pages.
- Lin, Anching and Lee, Sang-Myung, 1972, A Hele-Shaw Model Study of Seepage Flow Through the Causeway of the Great Salt Lake. *1st Annual Conference on the Great Salt Lake and Utah's Water Resources*, sponsored by the Utah Section of American Water Resources Association, 9 pages.
- Polubarinova-Kochina, P. Ya., 1962, *The Theory of Ground-Water Movement*, translated by R. J. M. DeWiest, Princeton University Press, Pages 431-448.
- Todd, D. K., 1954, Unsteady Flow in Porous Media by Means of a Hele-Shaw Viscous Fluid Model, *Trans. American Geophysical Union*, 35 (b), Pages 905-916.